EEC S205000: Linear Algebra **Spring 2020**
Department of Electrical Engineering
National Tsing Hua University

Homework #1

Coverage: Chapter 1-2

Due date: 1 April, 2020

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Notice:
1. Please hand in your answer sheets by yourself before 12:00 of the due date. No late homework will be accepted.
2. Please justify your answers with clear, logical and solid reasoning or proofs.
3. Please do the homework independently by yourself. However, you may discuss with someone else but copyied homework is not allowed. This will show your respect toward the academic integrity.

Problem 1. (10 points) Show that any linear combination of $\begin{bmatrix} 1 \\ 3/2 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 3 \\ 6 \end{bmatrix}$ is also a linear combination of $\begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$.

Problem 2. (10 points) Let $\mathbf{A}$ be a square matrix. Is it true that $(\mathbf{A} - 3\mathbf{I})(\mathbf{A} + 2\mathbf{I}) = \mathbf{0}$ implies $\mathbf{A} = 3\mathbf{I}$ or $\mathbf{A} = -2\mathbf{I}$? Why?

Problem 3. (10 points) Suppose $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$ are invertible matrices and $\mathbf{X}$ is a matrix such that $\mathbf{A} = \mathbf{BXC}$. Prove that $\mathbf{X}$ is invertible.

Problem 4. (15 points) Let $\mathbf{u} = \begin{bmatrix} 1 \\ -1 \\ -1 \\ 1 \\ 1 \end{bmatrix}$. Find $(\mathbf{I}_5 + 2\mathbf{u}\mathbf{u}^{\mathrm{T}})(\mathbf{I}_5 + \mathbf{u}\mathbf{u}^{\mathrm{T}})^{-1}\mathbf{u}$, where $\mathbf{I}_5$ is the identity matrix in $\mathbb{R}^{5\times 5}$.

Problem 5. (15 points) Let matrices $\mathbf{A} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -2 & 3 & 0 & 0 \\ 0 & -4 & 5 & 0 \\ 0 & 0 & -6 & 7 \end{bmatrix}$, $\mathbf{I} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ and $\mathbf{B} = (\mathbf{I}+\mathbf{A})^{-1}(\mathbf{I}-\mathbf{A})$. Find the matrix $(\mathbf{I}+\mathbf{B})^{-1}$.

Problem 6. (10 points) Consider the matrix $\mathbf{A} = \begin{bmatrix} 1 & a & 0 & 0 \\ 0 & 1 & b & 0 \\ 0 & 0 & 1 & c \\ 0 & 0 & 0 & 1 \end{bmatrix}$. Find the inverse of $\mathbf{A}$ using the Gauss-Jordan elimination method.

Problem 7. (20 points) Consider $\mathbf{K} = \begin{bmatrix} 2 & 1 & 1 \\ 4 & 3 & -1 \\ -6 & -2 & a \end{bmatrix}$ where $a \in \mathbb{R}$.

(i) Compute the LU factorizaion of the matrix $\mathbf{K}$.

(ii) Find condition on a such that LU has three pivots.

Problem 8. (10 points) Let $\mathbf{v}_1 = \begin{bmatrix} \alpha - 3 \\ 2 + \alpha \end{bmatrix}$ and $\mathbf{v}_2 = \begin{bmatrix} 2 - \alpha \\ \alpha + 3 \end{bmatrix}$ where $\alpha \in \mathbb{R}$. Find the values of α such that $\mathbf{v}_1$ and $\mathbf{v}_2$ are linearly independent.